

Probing Smearing Effect by Point-Like Graviton in Plane-Wave Matrix Model

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Abstract

We investigate the interaction between flat membrane and point-like graviton in the plane-wave matrix model. The one-loop effective potential in the large distance limit is computed and is shown to be of r^{-3} type where r is the distance between two objects. This type of interaction has been interpreted as the one incorporating the smearing effect due to the configuration of flat membrane in plane-wave background. Our result supports this interpretation and provides one more evidence about it.

Keywords : pp-wave, BMN matrix model, membrane

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1 Introduction

The plane-wave or BMN matrix model [1] is a model for the microscopic description of M-theory in the so called plane-wave background in the framework of the discrete light cone quantization (DLCQ). The plane-wave background [2] is $SO(3) \times SO(6)$ symmetric and given by

$$ds^2 = -2dx^+dx^- - \left(\sum_{i=1}^3 \left(\frac{\mu}{3} \right)^2 (x^i)^2 + \sum_{a=4}^9 \left(\frac{\mu}{6} \right)^2 (x^a)^2 \right) (dx^+)^2 + \sum_{I=1}^9 (dx^I)^2 ,$$

$$F_{+123} = \mu , \tag{1.1}$$

with the index notation $I = (i, a)$. This background is maximally supersymmetric and obtained by taking the Penrose limit to the eleven-dimensional AdS type geometries [3].

From the structural point of view, the plane-wave matrix model is a mass deformation of the matrix model in flat spacetime, the BFSS matrix model [4]. Compared to the BFSS matrix model, one distinguished feature of the plane-wave matrix model is that the supersymmetric fuzzy sphere membrane with finite size appears from the vacuum structure [1, 5]. Although it is a configuration of membrane, it has been interpreted as a graviton, or more precisely a giant graviton because it has a size. The presence of the fuzzy sphere membrane has led to a lot of works studying its nature from various viewpoints [5]-[11]. In the study of dynamical aspect, it has been shown that the fuzzy sphere behaves indeed like a graviton, and evidences about its interpretation as a giant graviton have been accumulated [12]-[20]. The thermodynamical aspect of fuzzy sphere has also been considered in [21]-[25]. Upon a proper circle compactification, the plane-wave matrix model leads to the matrix string theory, which is related in the infrared limit to the free string theory in ten-dimensional plane wave background [26]-[30]. This string theory contains fuzzy spheres in its spectrum, whose various aspects also have been studied [31]-[34].

Among the studies on fuzzy sphere, an interesting result has been obtained in the investigation of the interaction between fuzzy sphere and flat membrane by one of the present authors [20]. At one-loop level, the effective potential has been calculated in the large r limit, where r is the distance between two objects. Because the fuzzy sphere can be regarded as a point-like object at large r , the leading interaction potential was expected to be of r^{-5} type based on the result from the BFSS matrix model [35]. However, the leading interaction has turned out to be of r^{-3} type rather than r^{-5} type. The interpretation for this unexpected result was as follows: If the supersymmetric flat membrane is placed in the $SO(6)$ symmetric space of the plane-wave then it spans and spins in four dimensional subspace basically due to the nature of plane-wave background. As a result, two more ex-

tra dimensions are required for its configuration. This fact is reflected in the interaction potential as the delocalization or smearing effect. Actually, the smearing effect has been already reported in the supergravity side [36, 37]. In the plane-wave background, it has been observed that some supergravity solutions show the delocalization or smearing of branes in some directions.

Although the interpretation of the interaction in terms of the smearing effect is interesting, it has been given from just one example and one may wonder whether we can give the same interpretation in other cases involving the flat membrane. In this paper, we will study the interaction from another configuration for the purpose of checking the previous interpretation, and give one more evidence about the smearing effect due to the flat membrane in the plane-wave matrix model. The configuration we will take is composed of one point-like graviton and one flat membrane, each of which is supersymmetric. Two objects are kept apart on a plane in the $SO(3)$ symmetric space. We note that the situation is different from the previous case [20] where one fuzzy sphere is separated from one flat membrane on a plane in the $SO(6)$ symmetric space. Actually, this gives the reason why we take the point-like graviton as the object interacting with the flat membrane. Basically, the interaction between supersymmetric objects is our concern. Contrary to the fuzzy sphere, the point-like graviton can be supersymmetric even if it has a motion in the $SO(3)$ symmetric space [10].

The organization of this paper is as follows. In the next section, we will give a brief introduction to the plane-wave matrix model. In Sec. 3, the background configuration composed of flat membrane and point-like graviton is presented. In Sec. 4, the formal one-loop path integration of the plane-wave matrix model around the background configuration of Sec. 3 is performed. From the result of path integration, the one-loop effective potential is obtained in Sec. 5. Finally, the conclusion follows in Sec. 6.

2 Plane-wave matrix model

The plane-wave matrix model is basically composed of two parts. One part is the usual matrix model based on eleven-dimensional flat space-time, that is, the flat space matrix model [4], and another is a set of terms depending on μ and reflecting the structure of the maximally supersymmetric eleven dimensional plane-wave background, Eq. (1.1). Its action

is

$$S_{pp} = \int dt \text{Tr} \left(\frac{1}{2} D_t X^I D_t X^I + \frac{1}{4} ([X^I, X^J])^2 + i \Theta^\dagger D_t \Theta - \Theta^\dagger \gamma^I [\Theta, X^I] \right. \\ \left. - \frac{1}{2} \left(\frac{\mu}{3} \right)^2 (X^i)^2 - \frac{1}{2} \left(\frac{\mu}{6} \right)^2 (X^a)^2 - i \frac{\mu}{3} \epsilon^{ijk} X^i X^j X^k - i \frac{\mu}{4} \Theta^\dagger \gamma^{123} \Theta \right), \quad (2.1)$$

where D_t is the covariant derivative with the gauge field A ,

$$D_t = \partial_t - i[A, \] , \quad (2.2)$$

and γ^I is the 16×16 $SO(9)$ gamma matrices.

In matrix model, various objects, like branes and graviton, are realized by the classical solutions of the equations of motion for the matrix field, which are derived from Eq. (2.1) as follows:

$$\ddot{X}^i = -[[X^i, X^I], X^I] - \left(\frac{\mu}{3} \right)^2 X^i - i\mu \epsilon^{ijk} X^j X^k , \\ \ddot{X}^a = -[[X^a, X^I], X^I] - \left(\frac{\mu}{6} \right)^2 X^a , \quad (2.3)$$

where the over dot implies the time derivative ∂_t . Here, since the object that we are concerned about is purely bosonic, only the equations of motion for the bosonic field have been presented. For given objects, the dynamics between them is studied by expanding the matrix model action around the corresponding classical solution and performing the path integration. Let us denote the classical solution or the background configuration by B^I , and split the matrix quantities into as follows:

$$X^I = B^I + Y^I, \quad A = 0 + A, \quad \Theta = 0 + \Psi. \quad (2.4)$$

Then Y^I , A and Ψ are the quantum fluctuations around the background configuration, which are the fields subject to the path integration. We note that the gauge field may also have non-trivial classical configuration. However, it is simply set to zero in this paper because the objects we are interested in do not generate any background gauge field.

In taking into account the quantum fluctuations, we should recall that the matrix model itself is a gauge theory. This implies that the gauge fixing condition should be specified before proceed further. In this paper, we take the background field gauge which is usually chosen in the matrix model calculation,

$$D_\mu^{\text{bg}} A_\mu^{\text{qu}} \equiv D_t A + i[B^I, X^I] = 0. \quad (2.5)$$

Then the corresponding gauge-fixing S_{GF} and Faddeev-Popov ghost S_{FP} terms are given by

$$S_{\text{GF}} + S_{\text{FP}} = \int dt \text{Tr} \left(-\frac{1}{2} (D_\mu^{\text{bg}} A_\mu^{\text{qu}})^2 - \bar{C} \partial_t D_t C + [B^I, \bar{C}][X^I, C] \right). \quad (2.6)$$

Now by inserting the decomposition of the matrix fields (2.4) into Eqs. (2.1) and (2.6), we get the gauge fixed plane-wave action S ($\equiv S_{pp} + S_{GF} + S_{FP}$) expanded around the classical background B^I . The resulting action is read as

$$S = S_0 + S_2 + S_3 + S_4 , \quad (2.7)$$

where S_k represents the action of order k with respect to the quantum fluctuations and, for each k , its expression is

$$\begin{aligned} S_0 &= \int dt \text{Tr} \left[\frac{1}{2} (\dot{B}^I)^2 - \frac{1}{2} \left(\frac{\mu}{3} \right)^2 (B^i)^2 - \frac{1}{2} \left(\frac{\mu}{6} \right)^2 (B^a)^2 + \frac{1}{4} ([B^I, B^J])^2 - i \frac{\mu}{3} \epsilon^{ijk} B^i B^j B^k \right] , \\ S_2 &= \int dt \text{Tr} \left[\frac{1}{2} (\dot{Y}^I)^2 - 2i \dot{B}^I [A, Y^I] + \frac{1}{2} ([B^I, Y^J])^2 + [B^I, B^J] [Y^I, Y^J] - i \mu \epsilon^{ijk} B^i Y^j Y^k \right. \\ &\quad \left. - \frac{1}{2} \left(\frac{\mu}{3} \right)^2 (Y^i)^2 - \frac{1}{2} \left(\frac{\mu}{6} \right)^2 (Y^a)^2 + i \Psi^\dagger \dot{\Psi} - \Psi^\dagger \gamma^I [\Psi, B^I] - i \frac{\mu}{4} \Psi^\dagger \gamma^{123} \Psi \right. \\ &\quad \left. - \frac{1}{2} \dot{A}^2 - \frac{1}{2} ([B^I, A])^2 + \dot{\bar{C}} \dot{C} + [B^I, \bar{C}] [B^I, C] \right] , \\ S_3 &= \int dt \text{Tr} \left[-i \dot{Y}^I [A, Y^I] - [A, B^I] [A, Y^I] + [B^I, Y^J] [Y^I, Y^J] + \Psi^\dagger [A, \Psi] \right. \\ &\quad \left. - \Psi^\dagger \gamma^I [\Psi, Y^I] - i \frac{\mu}{3} \epsilon^{ijk} Y^i Y^j Y^k - i \dot{\bar{C}} [A, C] + [B^I, \bar{C}] [Y^I, C] \right] , \\ S_4 &= \int dt \text{Tr} \left[-\frac{1}{2} ([A, Y^I])^2 + \frac{1}{4} ([Y^I, Y^J])^2 \right] . \end{aligned} \quad (2.8)$$

3 Background configuration

In this section, we set up the background configuration corresponding to one flat membrane and one graviton, and discuss about the perturbation theory around it.

Since the background is composed of two objects, the matrices representing the background have the 2×2 block diagonal form as

$$B^I = \begin{pmatrix} B_{(1)}^I & 0 \\ 0 & B_{(2)}^I \end{pmatrix} \quad (3.1)$$

where $B_{(s)}^I$ with $s = 1, 2$ are $N_s \times N_s$ matrices. If B^I are taken to be $N \times N$ matrices, then $N = N_1 + N_2$.

The first object represented by $B_{(1)}^I$ is taken to be the flat membrane found in [8]. It is 1/8-BPS object, and spans and spins in four dimensional subspace of the $SO(6)$ symmetric space as

$$\begin{aligned} B_{(1)}^4 &= Q \cos(\mu t/6) , & B_{(1)}^6 &= Q \sin(\mu t/6) , \\ B_{(1)}^5 &= P \cos(\mu t/6) , & B_{(1)}^7 &= P \sin(\mu t/6) , \end{aligned} \quad (3.2)$$

where $N_1 \times N_1$ matrices, Q and P , satisfy

$$[Q, P] = i\sigma, \quad (3.3)$$

with a small constant parameter σ . We note that, in order to describe the flat membrane properly, the size of the matrix should be infinite. In what follows, N_1 is thus implicitly taken to be infinite. Now, from this somewhat complicated configuration, we see that, at $t = 0$, the flat membrane is placed in x^4 - x^5 plane, and, as time goes by, one axis along x^4 rotates in x^4 - x^6 plane while another axis along x^5 rotates in x^5 - x^7 plane. At this point, one may notice that the configuration looks quite strange since the membrane of infinite size is spinning. However, this kind of situation may be understood as the one due to the choice of the coordinate system for representing the plane-wave background (1.1), and may disappear by going to a frame where the membrane looks like static one. Indeed, as we will see, it is possible to take a certain frame where the whole background configuration looks like static one.

The graviton is the second object $B_{(2)}^I$, which is represented by 1×1 matrix and taken to rotate in x^1 - x^2 as follows:

$$B_{(2)}^1 = r \cos(\mu t/3), \quad B_{(2)}^2 = r \sin(\mu t/3), \quad (3.4)$$

which is 1/2-BPS object as shown in [10]. Thus the graviton is placed in the transverse space of the flat membrane with distance r .

For the background configuration (3.1) given above, the classical value of the action S_0 is evaluated as $S_0/T = -\frac{1}{2}N_1\sigma^2$ where $T = 12\pi/\mu$ is the period of motion for the membrane. Here, since the motion is periodic, we have considered the action per one period. As for the graviton, the classical action simply vanishes.

From now on, what we are going to do is the computation of the one-loop correction to the classical action S_0 , that is, to the background, (3.2) and (3.4), due to the quantum fluctuations via the path integration of the quadratic action S_2 , and obtain the one-loop effective action Γ_{eff} or the effective potential V_{eff} . But, in order to justify the one-loop computation, it should be made clear that S_3 and S_4 of Eq. (2.8) can be regarded as perturbations. For this purpose, following [5], we rescale the fluctuations and parameters as

$$\begin{aligned} A &\rightarrow \mu^{-1/2}A, \quad Y^I \rightarrow \mu^{-1/2}Y^I, \quad C \rightarrow \mu^{-1/2}C, \quad \bar{C} \rightarrow \mu^{-1/2}\bar{C}, \\ r &\rightarrow \mu r, \quad t \rightarrow \mu^{-1}t, \quad Q \rightarrow \mu Q, \quad P \rightarrow \mu P, \quad \sigma \rightarrow \mu^2\sigma. \end{aligned} \quad (3.5)$$

Under this rescaling, the powers of μ are factored out from the action S in the background (3.2) and (3.4) as

$$S = \mu^3 S_0 + S_2 + \mu^{-3/2} S_3 + \mu^{-3} S_4, \quad (3.6)$$

where S_0 , S_2 , S_3 and S_4 do not have μ dependence. Obviously, this form of the action ensures us that, in the large μ limit, S_3 and S_4 can be treated as perturbations and the one-loop computation gives the sensible result.

Based on the structure of (3.1), we now write the quantum fluctuations in the 2×2 block matrix form as follows:

$$\begin{aligned} A &= \begin{pmatrix} 0 & \Phi^0 \\ \Phi^{0\dagger} & 0 \end{pmatrix}, \quad Y^I = \begin{pmatrix} 0 & \Phi^I \\ \Phi^{I\dagger} & 0 \end{pmatrix}, \quad \Psi = \begin{pmatrix} 0 & \chi \\ \chi^\dagger & 0 \end{pmatrix}, \\ C &= \begin{pmatrix} 0 & C \\ C^\dagger & 0 \end{pmatrix}, \quad \bar{C} = \begin{pmatrix} 0 & \bar{C} \\ \bar{C}^\dagger & 0 \end{pmatrix}. \end{aligned} \quad (3.7)$$

Although we denote the block off-diagonal matrices for the ghosts by the same symbols with those of the original ghost matrices, there will be no confusion since $N \times N$ matrices will never appear from now on. The reason why the block-diagonal parts are not considered is that they do not give any effect on the interaction between two objects at least at the one-loop level.

4 One-loop quantum fluctuations

In this section, we perform the path integration for the quadratic action, S_2 , around the classical background (3.1) with (3.2) and (3.4). We will state only the formal results whose actual evaluation will be described in the next section.

The quadratic action is largely composed of three decoupled sectors, which are bosonic, ghost, and fermionic sectors. In the path integration of each sector, the integration variables are matrices. For the actual evaluation of the path integration, it is usually useful to expand the matrix variables in a suitable matrix basis. Taking a matrix basis depends on the classical background under consideration.

For the present case where the flat membrane is involved, the commutation relation (3.3), the characteristic of the flat membrane provides a clue for the desired matrix basis. If we define

$$a = \frac{1}{\sqrt{2\sigma}}(Q + iP), \quad a^\dagger = \frac{1}{\sqrt{2\sigma}}(Q - iP), \quad (4.1)$$

then they satisfy the commutation relation

$$[a, a^\dagger] = 1, \quad (4.2)$$

and can be regarded as the annihilation and creation operators of simple harmonic oscillator. This fact allows us to express the fluctuations around the flat membrane in terms of the

oscillator states, on which a and a^\dagger act as

$$a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \quad (4.3)$$

Because the size of the membrane is given by N_1 , the oscillator number n runs from 0 to $N_1 - 1$, and hence has the upper bound. However, we note that actually there is no upper bound for n because N_1 should be infinite for the proper description of the flat membrane. So, we take N_1 to be infinite from now on.

The off-diagonal blocks of Eq. (3.7) are simply vectors because the point-like graviton background is described by 1×1 matrix. This fact and above consideration leads us now to take $|n\rangle$ as the matrix basis for the fluctuations. Then, in this matrix basis, each fluctuation matrix has the following type of matrix mode expansion

$$\Phi = \sum_{n=0}^{\infty} \phi_n |n\rangle. \quad (4.4)$$

This expansion allows us to reduce the path integration of the matrix variable to that of the mode ϕ_n .

4.1 Bosonic contributions

We first consider the path integration of bosonic fluctuations including also the ghost part.

The Lagrangian L_B for the purely bosonic fluctuations is given by

$$\begin{aligned} L_B = \text{Tr} \Big\{ & -|\dot{\Phi}^0|^2 + \Phi^{0\dagger}(r^2 + Q^2 + P^2)\Phi^0 \\ & + |\dot{\Phi}^I|^2 - \Phi^{I\dagger}(r^2 + Q^2 + P^2)\Phi^I - \frac{1}{3^2}|\Phi^i|^2 - \frac{1}{6^2}|\Phi^a|^2 \\ & - ir \sin(t/3)(\Phi^{3\dagger}\Phi^1 - \Phi^{1\dagger}\Phi^3) + ir \cos(t/3)(\Phi^{3\dagger}\Phi^2 - \Phi^{2\dagger}\Phi^3) \\ & + i\frac{2}{3}r \sin(t/3)(\Phi^{0\dagger}\Phi^1 - \Phi^{1\dagger}\Phi^0) - i\frac{2}{3}r \cos(t/3)(\Phi^{0\dagger}\Phi^2 - \Phi^{2\dagger}\Phi^0) \\ & - \frac{i}{3} \sin(t/6) [\Phi^{0\dagger}(Q\Phi^4 + P\Phi^5) - (\Phi^{4\dagger}Q + \Phi^{5\dagger}P)\Phi^0] \\ & + \frac{i}{3} \cos(t/6) [\Phi^{0\dagger}(Q\Phi^6 + P\Phi^7) - (\Phi^{6\dagger}Q + \Phi^{7\dagger}P)\Phi^0] \\ & - 2i\sigma \left[(\cos(t/6)\Phi^{4\dagger} + \sin(t/6)\Phi^{6\dagger}) (\cos(t/6)\Phi^5 + \sin(t/6)\Phi^7) \right. \\ & \left. - (\cos(t/6)\Phi^{5\dagger} + \sin(t/6)\Phi^{7\dagger}) (\cos(t/6)\Phi^4 + \sin(t/6)\Phi^6) \right] \Big\}. \quad (4.5) \end{aligned}$$

Because of the rotating background, the Lagrangian depends on time explicitly through trigonometric functions, which makes the path integration cumbersome. This kind of explicit time dependence can be removed by going to a certain frame where the background looks

like static. In fact, changing frame is natural, since the configuration in the original frame contains the rotating membrane of infinite size which is quite strange in physical sense. Then, in order to move to the desired frame, we take

$$\begin{aligned}\cos(t/3)\Phi^1 + \sin(t/3)\Phi^2 &\rightarrow \Phi^1, & -\sin(t/3)\Phi^1 + \cos(t/3)\Phi^2 &\rightarrow \Phi^2, \\ \cos(t/6)\Phi^4 + \sin(t/6)\Phi^6 &\rightarrow \Phi^4, & -\sin(t/6)\Phi^4 + \cos(t/6)\Phi^6 &\rightarrow \Phi^6, \\ \cos(t/6)\Phi^5 + \sin(t/6)\Phi^7 &\rightarrow \Phi^5, & -\sin(t/6)\Phi^5 + \cos(t/6)\Phi^7 &\rightarrow \Phi^7.\end{aligned}\quad (4.6)$$

Under these transformations, the above Lagrangian (4.5) becomes

$$\begin{aligned}L_B = \text{Tr} \Big[& -|\dot{\Phi}^0|^2 + \Phi^{0\dagger}(r^2 + Q^2 + P^2)\Phi^0 \\ & + |\dot{\Phi}^I|^2 - \Phi^{I\dagger}(r^2 + Q^2 + P^2)\Phi^I - \frac{1}{3^2}|\Phi^3|^2 - \frac{1}{6^2}|\Phi^8|^2 - \frac{1}{6^2}|\Phi^9|^2 \\ & + \frac{2}{3}(\Phi^{1\dagger}\dot{\Phi}^2 - \Phi^{2\dagger}\dot{\Phi}^1) + \frac{1}{3}(\Phi^{4\dagger}\dot{\Phi}^6 - \Phi^{6\dagger}\dot{\Phi}^4) + \frac{1}{3}(\Phi^{5\dagger}\dot{\Phi}^7 - \Phi^{7\dagger}\dot{\Phi}^5) \\ & - ir(\Phi^{2\dagger}\Phi^3 - \Phi^{3\dagger}\Phi^2) - \frac{i}{3}\Phi^{0\dagger}(2r\Phi^2 - Q\Phi^6 - P\Phi^7) \\ & + \frac{i}{3}(2r\Phi^{2\dagger} - \Phi^{6\dagger}Q - \Phi^{7\dagger}P)\Phi^0 - 2i\sigma(\Phi^{4\dagger}\Phi^5 - \Phi^{5\dagger}\Phi^4) \Big],\end{aligned}\quad (4.7)$$

which is obviously free of trigonometric functions having explicit time dependence.

The first observation for the Lagrangian (4.7) is that the matrix fields Φ^8 and Φ^9 are free and decoupled from other fields. This means that the path integration for these fields can be carried out immediately. If we use the matrix expansion Eq. (4.4) for each of Φ^8 and Φ^9 , and the relation

$$(Q^2 + P^2)|n\rangle = \sigma(2a^\dagger a + 1)|n\rangle = \sigma(2n + 1)|n\rangle, \quad (4.8)$$

which is derived from Eqs. (4.1), (4.2), and (4.3), then the result of the path integration is simply obtained as

$$\prod_{n=0}^{\infty} \det^{-2} \left(\Delta_n - \frac{1}{6^2} \right), \quad (4.9)$$

where we have defined

$$\Delta_n \equiv -\partial_t^2 - r^2 - \sigma(2n + 1). \quad (4.10)$$

Other matrix fields except for Φ^8 and Φ^9 are coupled to each other, and they should be taken into account as a whole. Let us denote the Lagrangian describing them as \widehat{L}_B , which is given by Eq. (4.7) with vanishing Φ^8 and Φ^9 . That is,

$$\widehat{L}_B = L_B|_{\Phi^8, \Phi^9=0}. \quad (4.11)$$

When each matrix variable is expanded in terms of Eq. (4.4), we can express this Lagrangian in terms of modes. By the way, as already noted in a previous work [20] done by one of the present authors, the terms linear in Q and P lead to coupling of modes with different oscillator number n because Q and P are linear combinations of the creation and annihilation operators as seen in Eq. (4.1). In order to avoid such mixing, we follow the prescription given in [35], and take the unitary transformation as follows:

$$\Phi^\pm \equiv \frac{1}{\sqrt{2}}(\Phi^4 \pm i\Phi^5), \quad \tilde{\Phi}^\pm \equiv \frac{1}{\sqrt{2}}(\Phi^6 \pm i\Phi^7). \quad (4.12)$$

Then the terms linear in Q and P become

$$\begin{aligned} & \text{Tr} [\Phi^{0\dagger}(Q\Phi^6 + P\Phi^7) - (\Phi^{6\dagger}Q + \Phi^{7\dagger}P)\Phi^0] \\ &= \sqrt{\sigma} \text{Tr} [\Phi^{0\dagger}(a\tilde{\Phi}^- + a^\dagger\tilde{\Phi}^+) - (\tilde{\Phi}^{+\dagger}a + \tilde{\Phi}^{-\dagger}a^\dagger)\Phi^0]. \end{aligned} \quad (4.13)$$

From this structure, we see that the matrix mode expansions for Φ^\pm and $\tilde{\Phi}^\pm$ should be taken as

$$\Phi^\pm = \sum_{n=\pm 1}^{\infty} \phi_n^\pm |n \mp 1\rangle, \quad \tilde{\Phi}^\pm = \sum_{n=\pm 1}^{\infty} \tilde{\phi}_n^\pm |n \mp 1\rangle, \quad (4.14)$$

if Φ^0 , Φ^1 , Φ^2 , and Φ^3 are taken to follow the expansion of Eq. (4.4). Here, the reason why Φ^\pm and $\tilde{\Phi}^\pm$ should have the same type of mode expansion is that they couple to each other with one time derivative.

In terms of the matrix mode expansions described above, the Lagrangian is now written without any mixing between different oscillator number n as

$$\hat{L}_B = \sum_{n=0}^{\infty} V_n^\dagger M_n V_n, \quad (4.15)$$

where $V_n = (\phi_n^0, \phi_n^1, \phi_n^2, \phi_n^3, \phi_n^+, \tilde{\phi}_n^+, \phi_n^-, \tilde{\phi}_n^-)^T$ and

$$M_n = \begin{pmatrix} -\Delta_n & 0 & -\frac{2i}{3}r & 0 & 0 & \frac{i}{3}\sqrt{\sigma n} & 0 & \frac{i}{3}\sqrt{\sigma(n+1)} \\ 0 & \Delta_n & \frac{2}{3}\partial_t & 0 & 0 & 0 & 0 & 0 \\ \frac{2i}{3}r & -\frac{2}{3}\partial_t & \Delta_n & -ir & 0 & 0 & 0 & 0 \\ 0 & 0 & ir & \Delta_n - \frac{1}{3^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta_n & \frac{1}{3}\partial_t & 0 & 0 \\ -\frac{i}{3}\sqrt{\sigma n} & 0 & 0 & 0 & -\frac{1}{3}\partial_t & \Delta_n + 2\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Delta_n & \frac{1}{3}\partial_t \\ -\frac{i}{3}\sqrt{\sigma(n+1)} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3}\partial_t & \Delta_n - 2\sigma \end{pmatrix}, \quad (4.16)$$

with the Δ_n defined in Eq. (4.10). One may notice that the summation over n starts from 0 while the minimum value of n in the mode expansions of Eq. (4.14) is +1 or -1. We

illuminate on this. The oscillator number n of ϕ_n^+ and $\tilde{\phi}_n^+$ starts from $+1$ while that of ϕ_n^- and $\tilde{\phi}_n^-$ starts from -1 . It is easy to see that the modes ϕ_{-1}^- and $\tilde{\phi}_{-1}^-$ are decoupled from other modes and form a subsystem, because all other modes do not have such oscillator number. As for the modes ϕ_n^+ and $\tilde{\phi}_n^+$, the absence of them at $n = 0$ seems to require an independent treatment of M_0 . However, let us suppose that these modes were present at the beginning. Then, the structure of M_0 shows that they would be decoupled from other modes and form a subsystem. Furthermore, the subsystem is exactly the same with that composed of ϕ_{-1}^- and $\tilde{\phi}_{-1}^-$. This indicates that the modes ϕ_{-1}^- and $\tilde{\phi}_{-1}^-$ can be symbolically identified with ϕ_0^+ and $\tilde{\phi}_0^+$. More precisely, $\phi_{-1}^- \rightarrow \tilde{\phi}_0^+$ and $\tilde{\phi}_{-1}^- \rightarrow \phi_0^+$, which can be inferred from M_0 . After all, all the modes can be taken to have the oscillator number starting from $n = 0$. We would like to note that similar situation also appears in the investigation of the interaction between fuzzy sphere and flat membrane [20].

The mode expanded Lagrangian (4.15) allows us to evaluate the path integral as

$$\prod_{n=0}^{\infty} \text{Det}^{-1} M_n, \quad (4.17)$$

where Det involves the matrix determinant as well as the usual functional one. Usually, for the one-loop effective action or potential, the diagonalization of the matrix M_n is preferable. However, it is not an easy task, basically due to the two constant terms $\pm 2\sigma$ appearing in the diagonal elements of the matrix. Fortunately, we do not need to have the fully factorized form of the determinant of M_n , since we are interested in the interaction in the long distance limit and hence the perturbative expansion in terms of the long distance ($r \gg 1$) is enough for our purpose. Then, after some algebraic manipulation, it turns out that the above formal result of the path integral is written as

$$\prod_{n=0}^{\infty} \text{Det}^{-1} M_n = \prod_{n=0}^{\infty} \det^{-1}(\Delta_n P_n) \cdot \det^{-1}(1 - E_n), \quad (4.18)$$

where the definition of Δ_n is given in Eq. (4.10) and the quantities inside the functional determinants are defined by

$$\begin{aligned} P_n &\equiv (\Delta_n - \frac{1}{3^2})(\Delta_n - a_{n+})(\Delta_n - a_{n-})(\Delta_n - b_{n+})(\Delta_n - b_{n-})(\Delta_n - c_{n+})(\Delta_n - c_{n-}), \\ E_n &\equiv \frac{r^2}{3^2} \frac{1}{P_n} (\Delta_n - b_{n+})(\Delta_n - b_{n-}) \left[(\Delta_n - a_{n+})(\Delta_n - a_{n-}) + \frac{4}{3^2} \sigma(2n+1)(\Delta_n - \frac{1}{3^2}) \right] \\ &\quad + \frac{2\sigma^2}{P_n} \Delta_n \left[(2\Delta_n - \frac{1}{3^2})(\Delta_n - \frac{1}{3^2})(\Delta_n - c_{n+})(\Delta_n - c_{n-}) - \frac{r^2}{3^4} (14\Delta_n - \frac{5}{3^2}) \right], \end{aligned} \quad (4.19)$$

with

$$\begin{aligned}
a_{n\pm} &\equiv \frac{1}{18} \pm \frac{1}{3} \sqrt{r^2 + \frac{1}{6^2}}, \\
b_{n\pm} &\equiv \frac{1}{18} \pm \frac{1}{3} \sqrt{r^2 + \sigma(2n+1) + \frac{1}{6^2}}, \\
c_{n\pm} &\equiv \frac{2}{3^2} \pm \frac{1}{3} \sqrt{3^2 r^2 + 4\sigma(2n+1) + \frac{4}{3^2}}.
\end{aligned} \tag{4.20}$$

We have considered the purely bosonic fluctuations. The remaining thing is the ghost part associated with the gauge fixing. The Lagrangian for the ghost part is

$$L_G = \text{Tr} \left[\dot{\bar{C}}^\dagger \dot{C} - \bar{C}^\dagger (r^2 + Q^2 + P^2) C + \dot{\bar{C}} \dot{C}^\dagger - (r^2 + Q^2 + P^2) \bar{C} C^\dagger \right]. \tag{4.21}$$

The path integration is carried out by taking the same procedure used for the bosonic part. If we denote the modes of the ghost variables C and \bar{C} as c_n and \bar{c}_n respectively, the Lagrangian in terms of modes is obtained as

$$L_G = \sum_{n=0}^{\infty} \left[\dot{\bar{c}}_n^* \dot{c}_n + \dot{\bar{c}}_n \dot{c}_n^* - (r^2 + \sigma(2n+1)) (\bar{c}_n^* c_n + \bar{c}_n c_n^*) \right], \tag{4.22}$$

whose path integration is straightforward and results in

$$\prod_{n=0}^{\infty} \det^2 \Delta_n. \tag{4.23}$$

Let us now summarize the results and give the full expression obtained in the bosonic and ghost part. Eq. (4.9) is the result of the path integral for the matrix fields, Φ^8 and Φ^9 . For other purely bosonic matrix fields, the path integral leads to Eq. (4.18). As for the ghost part, we have obtained Eq. (4.23). The multiplication of these results gives

$$\prod_{n=0}^{\infty} \det^{-2} \left(\Delta_n - \frac{1}{6^2} \right) \cdot \det^{-1} (\Delta_n^{-1} P_n) \cdot \det^{-1} (1 - E_n), \tag{4.24}$$

which is the contribution to the one-loop effective action from the bosonic and ghost part. Here, we note that one half of the ghost contribution remains in the final result. This means that one half of the unphysical gauge degrees of freedom is not canceled explicitly and hidden in the bosonic contribution. Of course, this will not give any trouble. The effect of the gauge degrees of freedom is canceled somehow by that of ghost in the actual evaluation of the functional determinant.

4.2 Fermionic contribution

Having considered the bosonic and ghost fluctuations, let us turn to the fermionic sector of the quadratic action. The Lagrangian is

$$L_F = \text{Tr} \left[i\chi^\dagger \dot{\chi} - \frac{i}{4} \chi^\dagger \gamma^{123} \chi - r\chi^\dagger (\gamma^1 \cos(t/3) + \gamma^2 \sin(t/3)) \chi \right. \\ \left. + \chi^\dagger (\gamma^4 \cos(t/6) + \gamma^6 \sin(t/6)) Q \chi + \chi^\dagger (\gamma^5 \cos(t/6) + \gamma^7 \sin(t/6)) P \chi \right], \quad (4.25)$$

where the matrix variable χ has been rescaled by a factor $1/\sqrt{2}$. As we have done in the previous subsection, we first go to the frame, where background configuration becomes static one, by taking the rotation

$$\chi \longrightarrow \Lambda \chi, \quad (4.26)$$

where $\Lambda = e^{-\frac{1}{6}t\gamma^{12}} e^{-\frac{1}{12}t\gamma^{46}} e^{-\frac{1}{12}t\gamma^{57}}$. In this frame, the Lagrangian becomes

$$L_F = \text{Tr} \chi^\dagger \left[i\partial_t - \frac{i}{4} \gamma^{123} - r\gamma^1 + \gamma^4 Q + \gamma^5 P - \frac{i}{12} (2\gamma^{12} + \gamma^{46} + \gamma^{57}) \right] \chi. \quad (4.27)$$

The above Lagrangian contains various products of gamma matrices, which may lead to some complexity in practical calculation. In order to reduce the possible complexity, we first note the fact that χ is in **16** of $SO(9)$, that is, $\gamma_{(9)}\chi = \chi$ where $\gamma_{(9)} = \gamma^1\gamma^2\cdots\gamma^9$. If we consider the operator measuring the chirality in the $SO(6)$ symmetric space as $\gamma_{(6)} = \gamma^4\gamma^5\gamma^6\gamma^7\gamma^8\gamma^9$, we see that $\gamma_{(9)} = \gamma^{123}\gamma_{(6)}$. This shows that, for a given eigenvalue of $\gamma_{(9)}$, the eigenvalue of γ^{123} is automatically determined by that of $\gamma_{(6)}$, or vice versa. In succession, because $\gamma_{(6)} = -\gamma^{46}\gamma^{57}\gamma^{89}$, the chiralities in 4-6, 5-7, and 8-9 planes determine the eigenvalue of γ^{123} . At this point, we observe that all the gamma matrices and their products in (4.27) commutes with γ^{89} . This means that the spinor components of χ with different eigenvalues of γ^{89} do not couple in the Lagrangian. That is, if we split χ in terms of the chirality in 8-9 plane as

$$\chi = \chi^{(+)} + \chi^{(-)}, \quad (4.28)$$

where $\chi^{(s)}$ ($s = \pm$) has eight independent components and satisfies

$$\gamma^{89}\chi^{(s)} = is\chi^{(s)}, \quad (4.29)$$

then the Lagrangian is decomposed into two independent systems as follows:

$$L_F = L_F^{(+)} + L_F^{(-)}. \quad (4.30)$$

Here, $L_F^{(s)}$ contains only $\chi^{(s)}$ and is given by

$$L_F^{(s)} = \text{Tr} \chi^{(s)\dagger} \left[i\partial_t + \frac{s}{4}\gamma^{46}\gamma^{57} - r\gamma^1 + \gamma^4 Q + \gamma^5 P - \frac{i}{12}(2\gamma^{12} + \gamma^{46} + \gamma^{57}) \right] \chi^{(s)}, \quad (4.31)$$

where we have used $\gamma^{123} = \gamma^{46}\gamma^{57}\gamma^{89}$ as explained above.

As one may notice, splitting into two independent systems described by $L_F^{(+)}$ and $L_F^{(-)}$ simplifies the problem, because each of them is a system of spinor with eight independent components unlike the original Lagrangian L_F containing sixteen component spinor. To make more tractable form of each system, let us now split $\chi^{(s)}$ in terms of the chiralities in 1-2, 4-6, and 5-7 planes as

$$\chi^{(s)} = \sum_{s_1, s_2, s_3 = \pm} \chi_{s_1 s_2 s_3}^{(s)}, \quad (4.32)$$

where s_1 , s_2 , and s_3 represent the eigenvalues of γ^{12} , γ^{46} , and γ^{57} , respectively. Then, the action of γ^{12} on $\chi_{s_1 s_2 s_3}^{(s)}$ is given by $\gamma^{12}\chi_{s_1 s_2 s_3}^{(s)} = i s_1 \chi_{s_1 s_2 s_3}^{(s)}$ and similarly for γ^{46} and γ^{57} .

Besides the products of gamma matrices, the presence of Q and P in the Lagrangian of Eq. (4.31) leads to the mixing of modes with different oscillator number n when the spinor matrix $\chi_{s_1 s_2 s_3}^{(s)}$ is expanded according to Eq. (4.4). As we have done in the bosonic case, such mixing problem is cured by taking an appropriate unitary transformation and then newly defined mode expansions for some variables. We first consider the following unitary transformation.

$$\begin{aligned} \zeta_1^\pm &\equiv \frac{1}{\sqrt{2}}(\gamma^4 \chi_{+++} \mp i\gamma^5 \chi_{+--}), \\ \zeta_2^\pm &\equiv \frac{1}{\sqrt{2}}(\gamma^4 \chi_{-+-} \mp i\gamma^5 \chi_{--+}), \\ \zeta_3^\pm &\equiv \frac{1}{\sqrt{2}}(\gamma^4 \chi_{-++} \mp i\gamma^5 \chi_{---}), \\ \zeta_4^\pm &\equiv \frac{1}{\sqrt{2}}(\gamma^4 \chi_{++-} \mp i\gamma^5 \chi_{+-+}), \end{aligned} \quad (4.33)$$

where the superscript (s) in the spinor variables has been omitted and its presence is implicit from now on. This unitary transformation is taken in such a way that the creation and annihilation operators a^\dagger and a defined in Eq. (4.1) appear independently in different terms. After the transformation, we find that ζ_1^\pm and ζ_4^\pm couple to each other as $-i\sqrt{2\sigma}\zeta_1^{\pm\dagger}a^\dagger\gamma^5\zeta_4^\pm + i\sqrt{2\sigma}\zeta_1^{\mp\dagger}a\gamma^5\zeta_4^\pm$ and its conjugation. ζ_2^\pm and ζ_3^\pm have the similar coupling. Like the case of Eq. (4.14), the structure of couplings leads us to take the mode expansions for ζ_{2m}^\pm and ζ_{3m}^\pm as

$$\zeta_2^\pm = \sum_{n=\mp 1}^{\infty} \zeta_{2n}^\pm |n \pm 1\rangle, \quad \zeta_4^\pm = \sum_{n=\mp 1}^{\infty} \zeta_{4n}^\pm |n \pm 1\rangle, \quad (4.34)$$

while ζ_1^\pm and ζ_3^\pm are taken to have the standard mode expansion as in Eq. (4.4). Now, based on these mode expansions, we see that the Lagrangian of Eq. (4.31) does not have any coupling between modes with different oscillator number, and is written as

$$L_F^{(s)} = \sum_{n=0}^{\infty} Z_n^{(s)\dagger} F_n^{(s)} Z_n^{(s)}, \quad (4.35)$$

where $Z_n^{(s)} = (\zeta_{1n}^+, \zeta_{1n}^-, \zeta_{2n}^+, \zeta_{2n}^-, \zeta_{3n}^+, \zeta_{3n}^-, \zeta_{4n}^+, \zeta_{4n}^-)^T$ and

$$F_n^{(s)} = \begin{pmatrix} K_1^{(s)} & 0 & D & \Gamma_n \\ 0 & K_2^{(s)} & \Gamma_n^\dagger & D \\ D & \Gamma_n & K_3^{(s)} & 0 \\ \Gamma_n^\dagger & D & 0 & K_4^{(s)} \end{pmatrix}. \quad (4.36)$$

The various quantities inside the matrix $F_n^{(s)}$ are 2×2 matrices and defined by

$$\begin{aligned} K_1^{(s)} &= \begin{pmatrix} i\partial_t - \frac{1}{4}s + \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & i\partial_t - \frac{1}{4}s + \frac{1}{6} \end{pmatrix}, & K_2^{(s)} &= \begin{pmatrix} i\partial_t + \frac{1}{4}s - \frac{1}{6} & 0 \\ 0 & i\partial_t + \frac{1}{4}s - \frac{1}{6} \end{pmatrix}, \\ K_3^{(s)} &= \begin{pmatrix} i\partial_t - \frac{1}{4}s - \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & i\partial_t - \frac{1}{4}s - \frac{1}{6} \end{pmatrix}, & K_4^{(s)} &= \begin{pmatrix} i\partial_t + \frac{1}{4}s + \frac{1}{6} & 0 \\ 0 & i\partial_t + \frac{1}{4}s + \frac{1}{6} \end{pmatrix}, \end{aligned} \quad (4.37)$$

and

$$\Gamma_n = \begin{pmatrix} 0 & -i\sqrt{2\sigma n}\gamma^5 \\ i\sqrt{2\sigma(n+1)}\gamma^5 & 0 \end{pmatrix}, \quad D = r \begin{pmatrix} \gamma^1 & 0 \\ 0 & \gamma^1 \end{pmatrix}. \quad (4.38)$$

We would like to note that, in writing the Lagrangian $L_F^{(s)}$ of Eq. (4.35), we have used the reasoning similar to that leading to L_B of Eq. (4.15), and symbolically identified $\zeta_{2,-1}^+$ and $\zeta_{4,-1}^+$ with $\zeta_{2,0}^-$ and $\zeta_{4,0}^-$ respectively. Thus, the summation for n starts from 0 for all modes.

The path integration for the Lagrangian $L_F^{(s)}$ is now evaluated as

$$\prod_{n=0}^{\infty} \text{Det} F_n^{(s)}. \quad (4.39)$$

This is of course not the final form, and we should compute the matrix determinant by exploiting the following matrix identity repeatedly.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & 1 \end{pmatrix} \begin{pmatrix} 1 & A^{-1}B \\ 0 & D - CA^{-1}B \end{pmatrix}. \quad (4.40)$$

After a little bit long manipulation, we see that $\text{Det} F_n^{(s)}$ turns out to be written as

$$\text{Det} F_n^{(s)} = \det Q_n \cdot \det(1 - R_n^{(s)}), \quad (4.41)$$

where we have defined the following quantities

$$Q_n = (\Delta_n - \hat{a}_{n+})(\Delta_n - \hat{a}_{n-})(\Delta_n - \hat{b}_{n+})(\Delta_n - \hat{b}_{n-}), \quad (4.42)$$

$$\begin{aligned} \hat{a}_{n\pm} &\equiv \frac{5}{48} - \frac{1}{6} \sqrt{r^2 + \frac{1}{6^2}} \pm \alpha_{n-}, \quad \hat{b}_{n\pm} \equiv \frac{5}{48} + \frac{1}{6} \sqrt{r^2 + \frac{1}{6^2}} \pm \alpha_{n+}, \\ \alpha_{n\pm} &\equiv \frac{1}{3} \sqrt{\frac{5}{2} r^2 + \frac{5}{4} \sigma(2n+1) + \frac{59}{576} + 9\sigma^2} \pm \frac{5}{12} \sqrt{r^2 + \frac{1}{6^2}}, \end{aligned} \quad (4.43)$$

$$\begin{aligned} R_n^{(s)} &\equiv \frac{1}{Q_n} \left[-\frac{1}{6^2} \left(r^2 + \frac{1}{3 \cdot 4^2} \right) \partial_t^2 + \frac{1}{9^2} \sigma(2n+1) \left(-\partial_t^2 - r^2 + \frac{5}{12^2} \right) \right. \\ &\quad + \frac{4}{9} \sigma^2 \left(-\partial_t^2 - r^2 - \frac{1}{12^2} \right) + \frac{s}{2 \cdot 9^2} \sigma(2n+1) i \partial_t \\ &\quad + \frac{s}{6} \left(\frac{1}{12} + \sqrt{r^2 + \frac{1}{6^2}} \right) (\Delta_n - \hat{a}_{n+})(\Delta_n - \hat{a}_{n-}) i \partial_t \\ &\quad \left. + \frac{s}{6} \left(\frac{1}{12} - \sqrt{r^2 + \frac{1}{6^2}} \right) (\Delta_n - \hat{b}_{n+})(\Delta_n - \hat{b}_{n-}) i \partial_t \right]. \end{aligned} \quad (4.44)$$

By using the above result for the functional determinant, we can give finally the full result of path integration for the fermionic fluctuations as

$$\begin{aligned} &\prod_{n=0}^{\infty} \text{Det} F_n^{(+)} \cdot \text{Det} F_n^{(-)} \\ &= \prod_{n=0}^{\infty} \det^2 Q_n \cdot \det(1 - R_n^{(+)}) \cdot \det(1 - R_n^{(-)}). \end{aligned} \quad (4.45)$$

5 Effective potential

We are now ready to compute the effective potential by using the formal path integral results obtained in the last section, Eqs. (4.24) and (4.45).¹ The multiplication of these results gives $\exp(i\Gamma_{\text{eff}})$, where Γ_{eff} is the one-loop effective action describing the interaction between the flat membrane and the graviton. The one-loop effective potential V_{eff} itself is related to the effective action via the relation $\Gamma_{\text{eff}} = -\int dt V_{\text{eff}}$.

Each of the functional determinants in Eqs. (4.24) and (4.45) is composed of two parts. One is the determinant whose argument has the factorized form and another with the argument of fractional form. We first consider the former one, and compute its contribution to

¹Almost all the computation in this section has been preformed by using *Mathematica*.

the effective potential in the limit of large distance. Then the relevant functional determinant is given by the multiplication of $\prod_{n=0}^{\infty} \det^{-2}(\Delta_n - \frac{1}{6^2}) \cdot \det^{-1}(\Delta_n^{-1} P_n)$ and $\prod_{n=0}^{\infty} \det^2 Q_n$. The calculation of this can be done without much difficulty, and the potential is read off as

$$\begin{aligned} \sum_{n=0}^{\infty} & \left[2\sqrt{m_n^2 + \frac{1}{6^2}} + \sqrt{m_n^2 + \frac{1}{3^2}} + \sqrt{m_n^2 + a_{n+}} + \sqrt{m_n^2 + a_{n-}} \right. \\ & + \sqrt{m_n^2 + b_{n+}} + \sqrt{m_n^2 + b_{n-}} + \sqrt{m_n^2 + c_{n+}} + \sqrt{m_n^2 + c_{n-}} - \sqrt{m_n^2} \\ & \left. - 2\sqrt{m_n^2 + \hat{a}_{n+}} - 2\sqrt{m_n^2 + \hat{a}_{n-}} - 2\sqrt{m_n^2 + \hat{b}_{n+}} - 2\sqrt{m_n^2 + \hat{b}_{n-}} \right], \end{aligned} \quad (5.1)$$

where $m_n^2 \equiv r^2 + \sigma(2n+1)$ and other quantities have been already defined in Eqs. (4.20) and (4.43). This expression contains the infinite sum over n , which may lead to the issue of convergence. However, the large n behavior of the summand can be shown to be of the order of $n^{-3/2}$, and thus the summation is well-defined and convergent. The sum over n itself can be performed by adopting the Euler-Maclaurin formula

$$\sum_{n=0}^{\infty} f(n) = \int_0^{\infty} dx f(x) + \frac{1}{2}f(0) - \frac{1}{12}f'(0) + \frac{1}{720}f'''(0) + \dots \quad (5.2)$$

which is valid when f and its derivatives vanish at infinity. After the summation, if we expand the resulting potential in terms of large r , we obtain

$$\frac{\sigma}{r} - \frac{67}{15552} \frac{1}{\sigma r} + \frac{1}{216} \frac{\sigma}{r^3} - \frac{215}{373248} \frac{1}{\sigma r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \quad (5.3)$$

We see that this potential is quite ridiculous in physical sense because of the terms which are inversely proportional to σ . Vanishing σ means the absence of the flat membrane in the background configuration, and hence it is expected that there is no potential. But, the above potential diverges when σ is set to zero. Therefore, for getting sensible result, the terms which are inversely proportional to σ must be canceled by contributions from other functional determinants. As it should be, we will see that such terms are absent at the final result.

We now turn to the effective potential which comes from the functional determinants with fractional argument. Let us first consider the contribution from the bosonic part, that is, $\prod_{n=0}^{\infty} \det^{-1}(1 - E_n)$ in Eq. (4.18). From the identity

$$\det^a(1 + A) = \exp[\text{atr} \ln(1 + A)] = \exp[\text{atr} A - \text{atr} A^2/2 + \dots] \quad (5.4)$$

where tr is the functional trace, we see that, through the simple power counting, the leading contribution to the effective potential at large distance is $i \sum_{n=0}^{\infty} \text{tr} E_n$. The trace calculation

of this is transformed to an integration in momentum space. After evaluating the integration, the Euler-Maclaurin formula (5.2) and the expansion in terms of large r then lead us to have the following contribution to the effective potential.

$$i \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} E_n = -\frac{\sigma}{r} + \frac{23}{3888} \frac{1}{\sigma r} + \frac{1}{216} \frac{\sigma}{r^3} + \frac{521}{933120} \frac{1}{\sigma r^3} + \mathcal{O}\left(\frac{1}{r^5}\right), \quad (5.5)$$

where ω is the conjugate variable of time t , and ∂_t^2 inside E_n is understood to be replaced by $-\omega^2$. This is the leading order result in the large distance limit. As a next step, one may consider the next to leading order contribution given by $(i/2) \sum_{n=0}^{\infty} \text{tr}(E_n)^2$. However, this and higher order contributions are turned out to give at most $\mathcal{O}(1/r^5)$, and hence they are not our concerns.

As for the fermionic part, the functional determinant with fraction argument is given by $\prod_{n=0}^{\infty} \det(1 - R_n^{(+)}) \cdot \det(1 - R_n^{(-)})$ of Eq. (4.45). The procedure of evaluating this is the same with that taken in the bosonic case. The leading order contribution to the effective potential in the large distance limit is obtained as

$$-i \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} [R_n^{(+)} + R_n^{(-)}] = -\frac{25}{15552} \frac{1}{\sigma r} - \frac{1}{27} \frac{\sigma}{r^3} + \frac{1}{38880} \frac{1}{\sigma r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \quad (5.6)$$

We note that, as can be noticed from the definition of $R_n^{(s)}$ in Eq. (4.44), the terms depending on the sign s do not enter at the leading order because of the cancellation between terms with opposite signs. Those terms contribute at the next to leading order. If we calculate the contribution from the next to leading order, then we get

$$-\frac{i}{2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} [(R_n^{(+)})^2 + (R_n^{(-)})^2] = -\frac{1}{124416} \frac{1}{\sigma r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \quad (5.7)$$

For higher order contributions, the power counting tells us that the potential is of the order of $\mathcal{O}(1/r^5)$. Thus, there is no need to consider higher orders in the large distance limit.

Up to now, we have obtained all the necessary results for giving the one-loop effective potential between graviton and flat membrane in the large distance limit. If we sum up the results obtained in Eqs. (5.3), (5.5), (5.6) and (5.7), then the final expression of the one-loop effective potential in the large distance limit becomes

$$V_{\text{eff}} = -\frac{1}{36} \frac{\sigma}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \quad (5.8)$$

We see that the leading order interaction is of r^{-3} type. Because there is no other contribution giving this kind of interaction term, it can be concluded that the leading interaction

of the effective potential is one-loop exact. As an important remark, we also see that the potential is completely sensible in a way that it does not contain terms which are inversely proportional to σ . All such terms have been canceled exactly with each other.

It should be noted that the effective potential (5.8) has the same type with that of the interaction potential [20] between fuzzy sphere (giant graviton) and flat membrane in plane-wave matrix model. In the context of the DLCQ M-theory in the flat spacetime, the interaction between graviton and flat membrane is usually expected to be of r^{-5} type, as explicitly illustrated in [35]. In [20], the result that the leading order interaction at large distance is of r^{-3} type rather than r^{-5} type has been interpreted as the one due to the smearing of flat membrane in two extra spatial dimensions. The potential (5.8) obtained in this paper supports this interpretation and provides one more evidence about it.

6 Conclusion

Motivated by the previous observation and interpretation in the study of the interaction between fuzzy sphere membrane and flat membrane, we have considered the configuration composed of one point-like graviton and one flat membrane, and investigated the interaction between them in the context of plane-wave matrix model.

At the one-loop level, the effective potential between graviton and flat membrane has been obtained, and its leading order interaction in the large distance limit has been shown to be of r^{-3} type. In [20], this type of interaction rather than r^{-5} type has been interpreted as the delocalization or smearing effect due to the configuration of the flat membrane which spans and spins in four dimensional space. Our final result (5.8) agrees well with this interpretation. Furthermore, it provides one more evidence for the smearing effect due to the configuration of flat membrane in plane-wave background.

Acknowledgments

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number 2005-0049409. The work of H.S. was also supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government with the grant No. KRF-2008-331-C00071 and R01-2008-000-21026-0.

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